

Direct Observation of the Quasi-Distribution Associated with the Wigner's Inequality

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Quantum correlations play a central role in quantum technologies, as well as in investigating quantum mechanics foundations. One way to characterize quantum correlations is by violating Wigner's inequality, which involves measurements of a set of incompatible (non-commuting) observables on entangled pairs [1,2]. Such a violation is due to the fact that the joint probability distribution for all observables—i.e., the one whose marginals reproduce the experimental outcome probabilities obtained from these measurements—can present negative values [3]. Estimating this distribution would provide deeper insight into the violation of the inequality arising from quantum correlations. However, due to its non-positive definiteness, its experimental characterization is usually deemed impossible.

Here we present a measurement scheme that allows the (quasi) direct experimental inference of such a distribution, realized via a quantum optical setup in which each photon of a polarization-entangled photon pair undergoes a sequence of two von Neumann-like measurements. For each measurement, the photon polarization is weakly coupled to a measurement probe, minimizing measurement back-action and enabling the sequential estimation of two non-commuting observables on both photons without collapsing their quantum state [4,5]. This allows inferring the joint (quasi-)probability distribution associated with the Wigner inequality violation (negative values included) without postselection or other additional operations, requiring only a proper calibration of the measurement apparatus.

This work leads to the first direct characterization of a quasi-probability distribution on a bipartite system, providing further advancement in understanding the nature of quantum correlations while also shedding new light on their characterization and exploitation.

References

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